



# Single-Trial Characterization of Evoked Responses

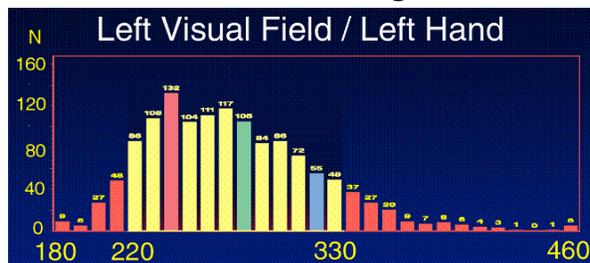
Bayesian Estimation and  
Differentially Variable Component Analysis

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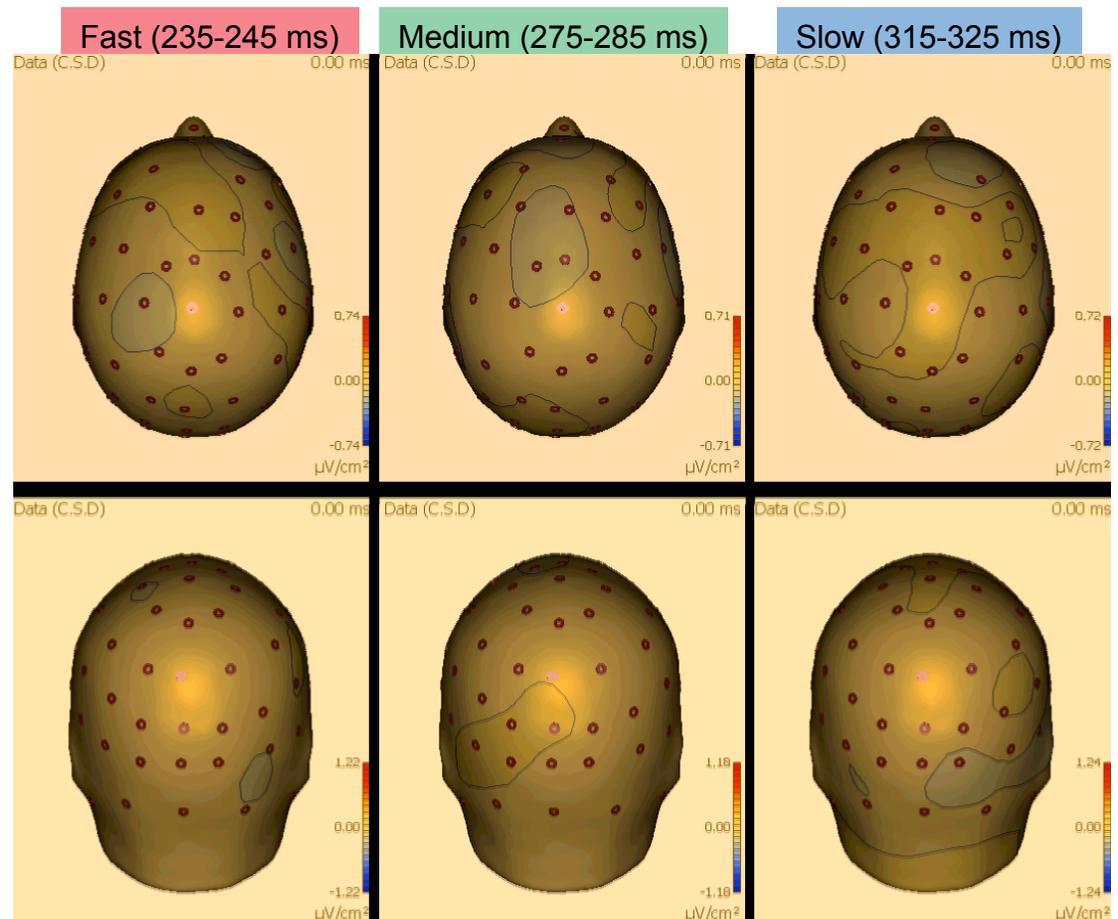
# Brains are Dynamic

Brains are dynamic, state dependent, and change over time.

Reaction Time Histogram



## Reaction Time



Movie courtesy of Clifford Saron.

Saron C.D. et al. 2003. In: *The Asymmetrical Brain*. Hugdahl, K and Davidson, R.J., Eds. Pp. 341-408.

# Intracranial ERP Recordings

## Intracranial Recordings

## Multi-Electrode Array

## Span Cortical Layers

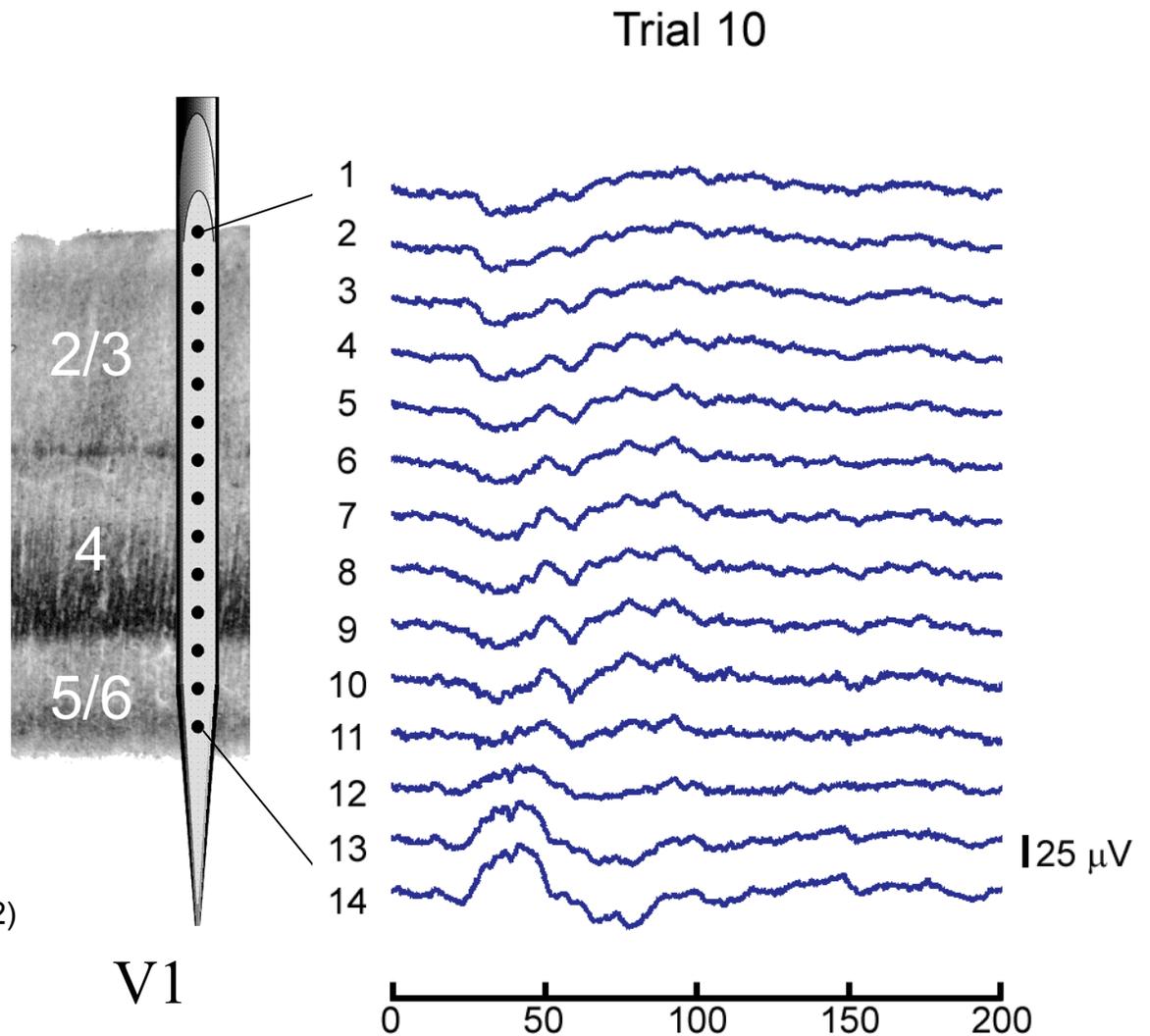
### In these data:

1. Evoked responses
2. No significant phase resetting

Shah AS, et al. 2004. In press: *Cereb Cortex*

3. Trial-to-trial variability

Truccolo WA, et al. 2002. *Clin Neurophysiol* 113(2)



# Averaging ERPs

Under what conditions is averaging single trial data appropriate?

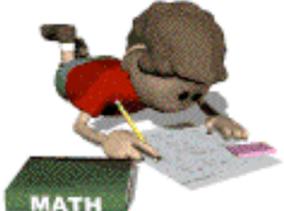
If we assume that the ERP is identical from trial-to-trial, and the data recorded from each trial consists of a sum of this ERP and ongoing background activity and noise, then our signal model for data recorded during trial  $r$  is

$$x_r(t) = s(t) + \eta_r(t)$$

Recorded Signal during the  $r^{\text{th}}$  trial

ERP waveform

Unpredictable Signal Component during the  $r^{\text{th}}$  trial (Ongoing Activity plus Noise)



# Maximum Likelihood Estimation

By assigning a Gaussian likelihood, we can find the most probable solution by maximizing

$$p(x(t) | s(t), I) = \left(2\pi\sigma^2\right)^{\frac{RT}{2}} \text{Exp} \left[ -\frac{1}{2\sigma^2} \sum_{r=1}^R \sum_{t=1}^T (x_r(t) - s(t))^2 \right]$$

Looking at the Log Probability

$$\text{Log } p = -\frac{1}{2\sigma^2} \sum_{r=1}^R \sum_{t=1}^T (x_r(t) - s(t))^2 + \text{const}$$

We take the derivative with respect to the ERP waveform at a time point  $q$

$$\frac{\partial \text{Log } p}{\partial s(q)} = \sigma^{-2} \sum_{r=1}^R (x_r(q) - s(q)) \equiv 0$$

Which setting to zero and solving for the ERP waveform gives  $\hat{s}(q) = \frac{1}{R} \sum_{r=1}^R x_r(q)$

Thus averaging the trials  
is optimal!

# Response Variability

However, the responses are not identical from trial to trial!

There is significant variability:

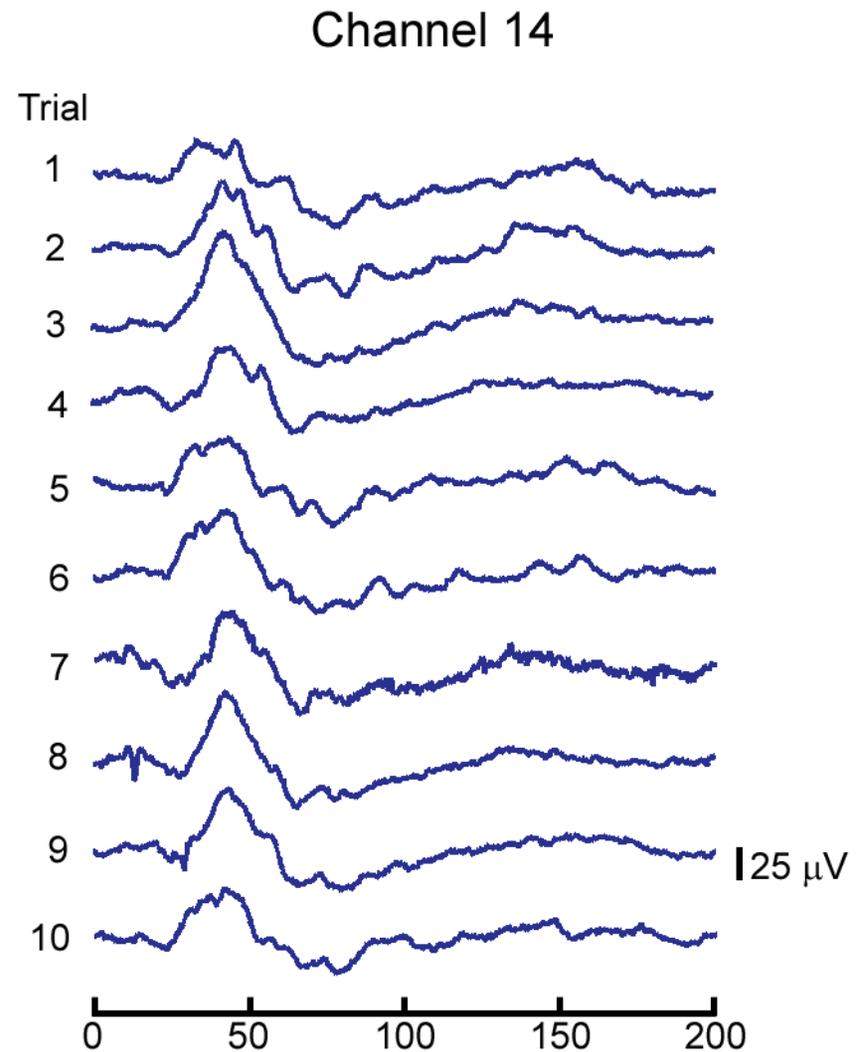
Amplitude variations

Latency variations

Waveshape variations

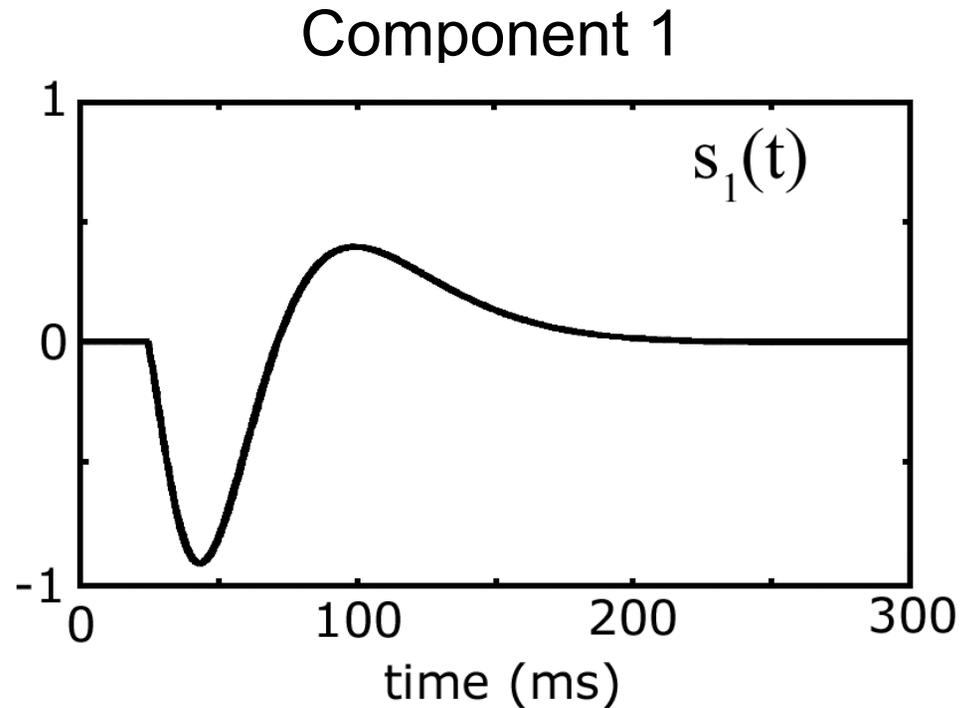
There are also multiple contributing components.

These variations are **INTERESTING.**



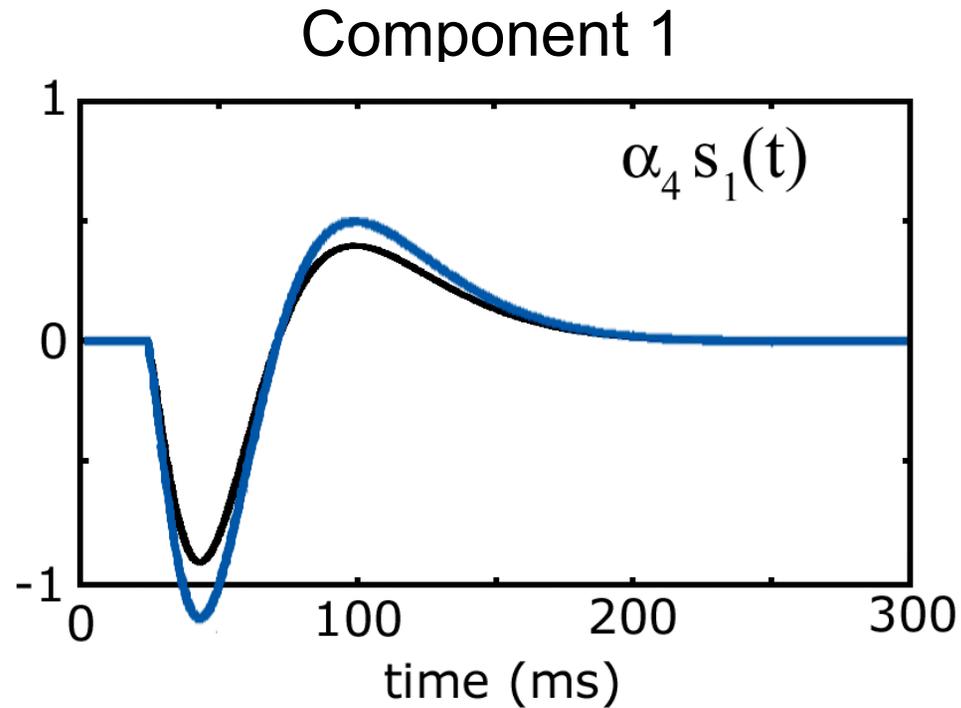
# Describing Trial-to-Trial Variability

We begin by describing the waveshape of a single component as a function of time.



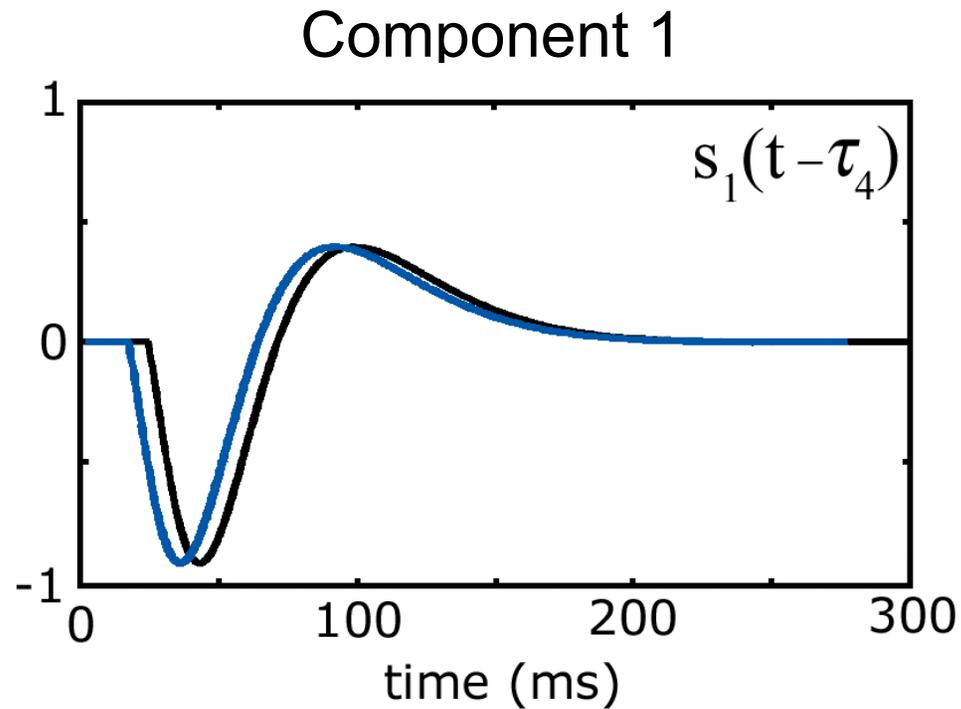
# Describing Amplitude Variability

Amplitude can vary from trial-to-trial



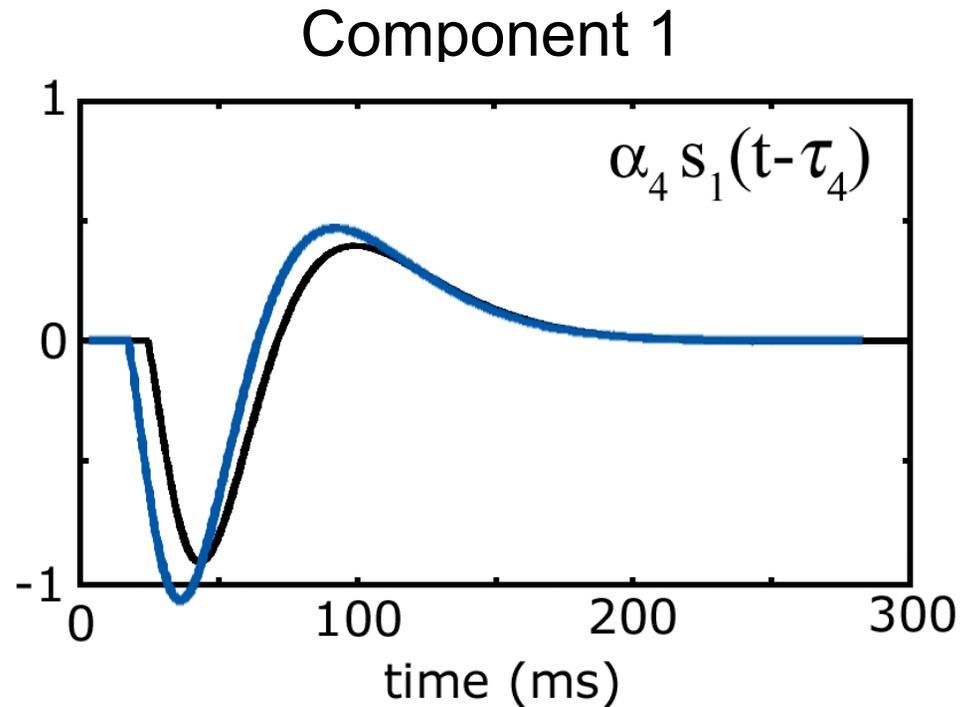
# Describing Latency Variability

Onset latency can vary from trial-to-trial



# Amplitude and Latency Variability

Now we describe amplitude and latency variability



# Multiple Component ERP Model

In this model we assume that each ERP component has its own stereotypic waveshape that can vary in both amplitude and latency from trial to trial. In addition, we allow for multiple channel recordings.

$$x_{mr}(t) = \sum_{n=1}^N C_{mn} \alpha_{nr} s_n(t - \tau_{nr}) + \eta_{mr}(t)$$

Recorded Signal in the  $m^{\text{th}}$  channel during the  $r^{\text{th}}$  trial

Coupling between the  $n^{\text{th}}$  source and the  $m^{\text{th}}$  channel

Amplitude of the  $n^{\text{th}}$  component during the  $r^{\text{th}}$  trial  
 $\langle \alpha_n \rangle = 1$

Latency of the  $n^{\text{th}}$  component during the  $r^{\text{th}}$  trial  
 $\langle \tau_n \rangle = 0$

Stereotypic waveform of the  $n^{\text{th}}$  component

# Learning the Parameters from Data

The goal is to use the recorded data to learn the model parameters.

We do this by designing a machine learning algorithm that uses Bayesian probability theory to find the most probable values of the model parameters, such as  $s(t)$ ,  $\alpha$ , and  $\tau$ , given the recorded data.

I have already shown how the most simple model leads to averaging.

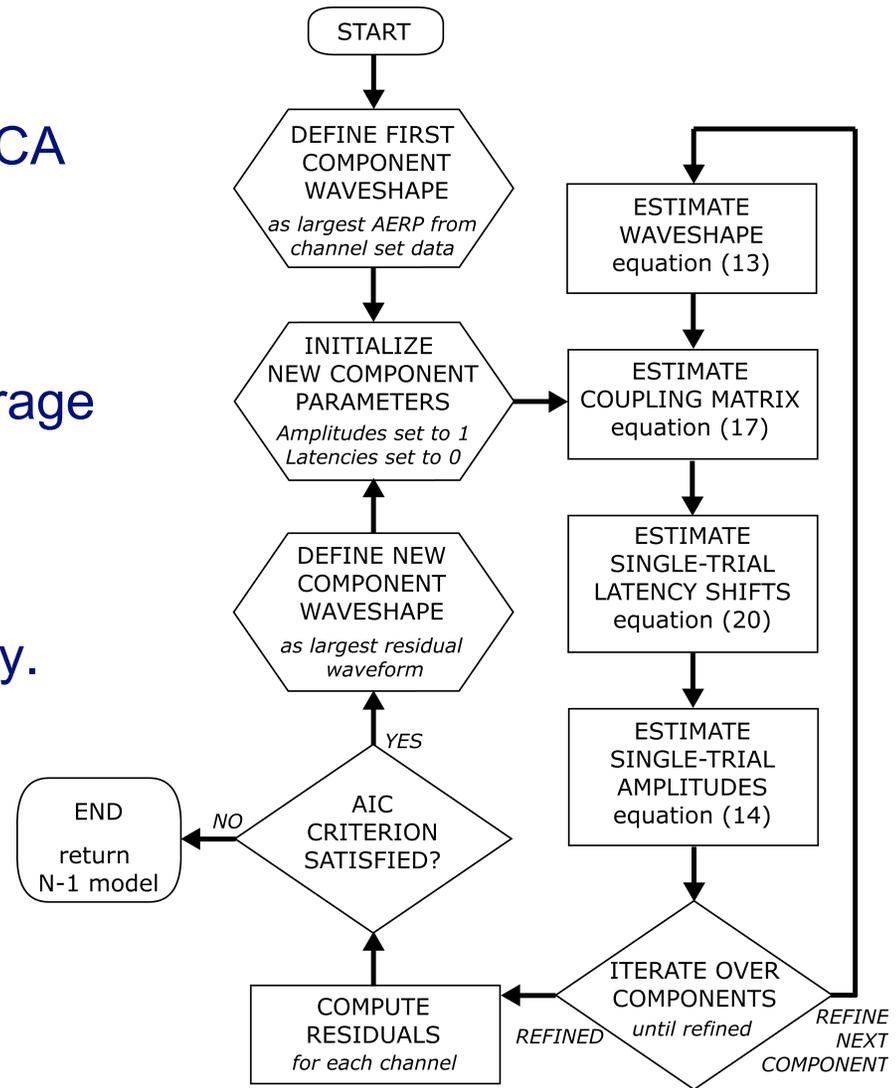
A mixing model that does not take into account variability, but assumes an amplitude density for the components leads to Independent Component Analysis (ICA). Knuth K.H. 1999. *ICA'99*.

# Differentially Variable Component Analysis

This is only one of a number of possible ways to implement the dVCA algorithm.

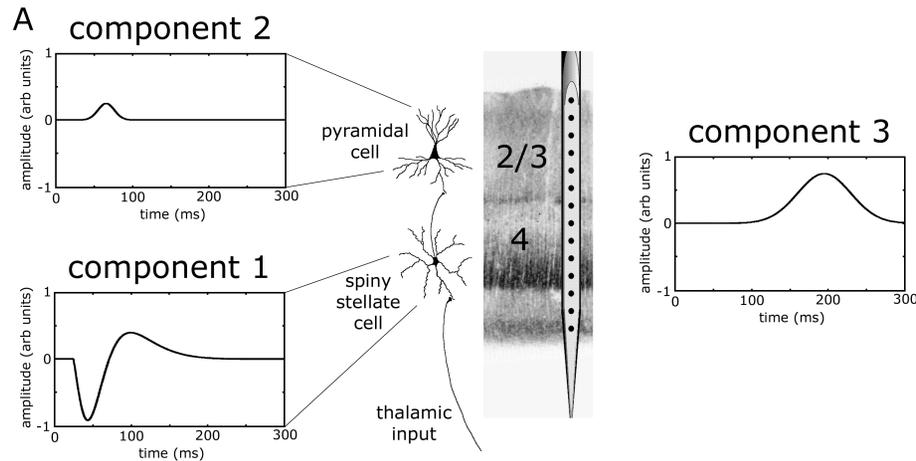
This technique begins with the average ERP and improves it.

Components are added sequentially.



# Simulations

# Synthetic Data

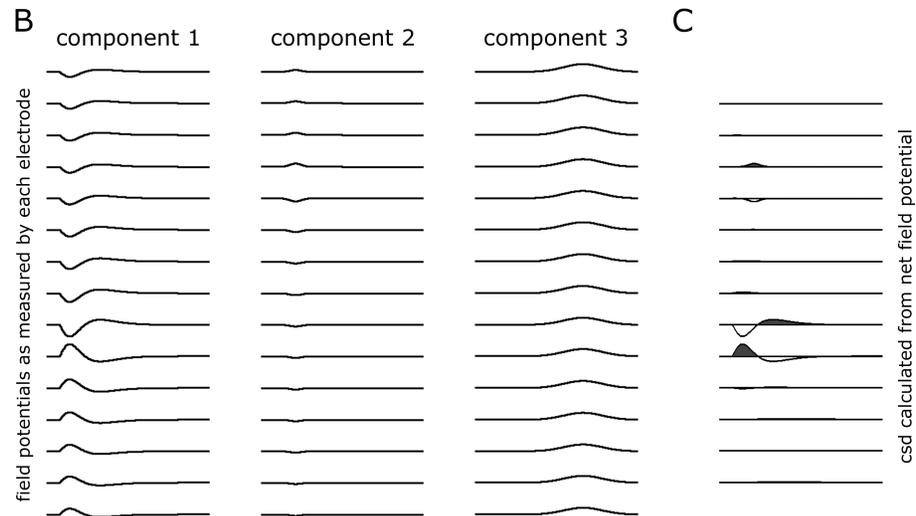


To characterize dVCA we created a synthetic dataset designed after the sources hypothesized from our earlier results in Macaque V1 in response to a Red Light Flash.

c1 Initial Layer IV response

c2 Granular response

c3 Far-field response



**A** Anatomical correlates

**B** Recorded waveshapes

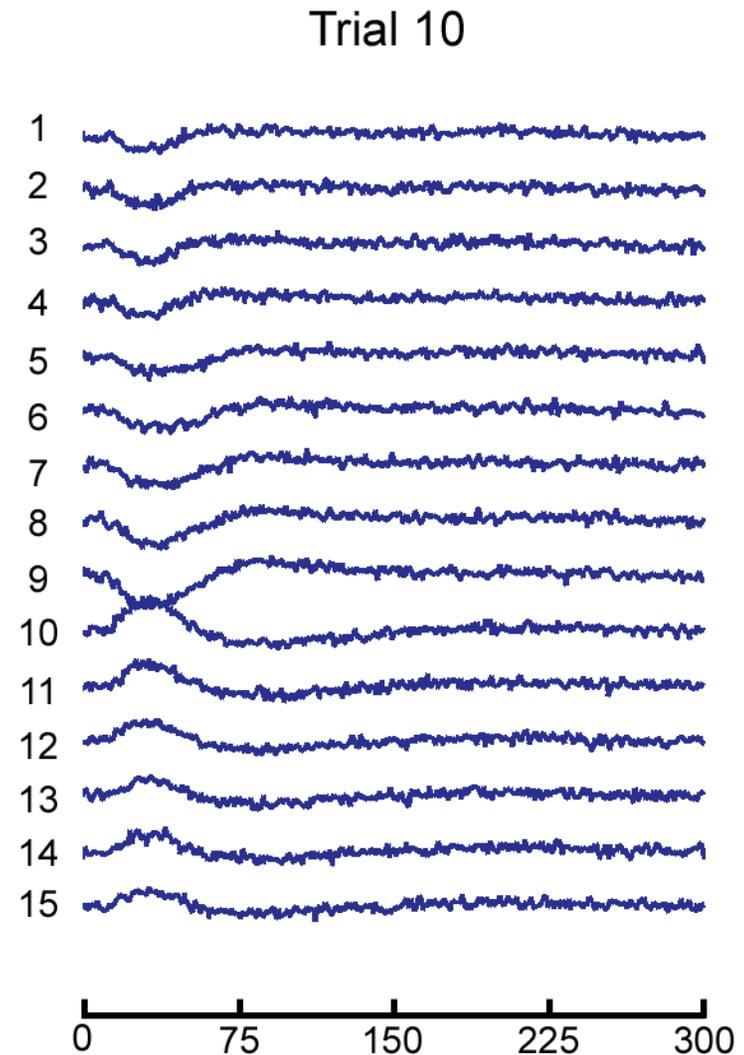
**C** Current Source Density (CSD)

# Multiple Synthetic Trials

## Synthetic Trials

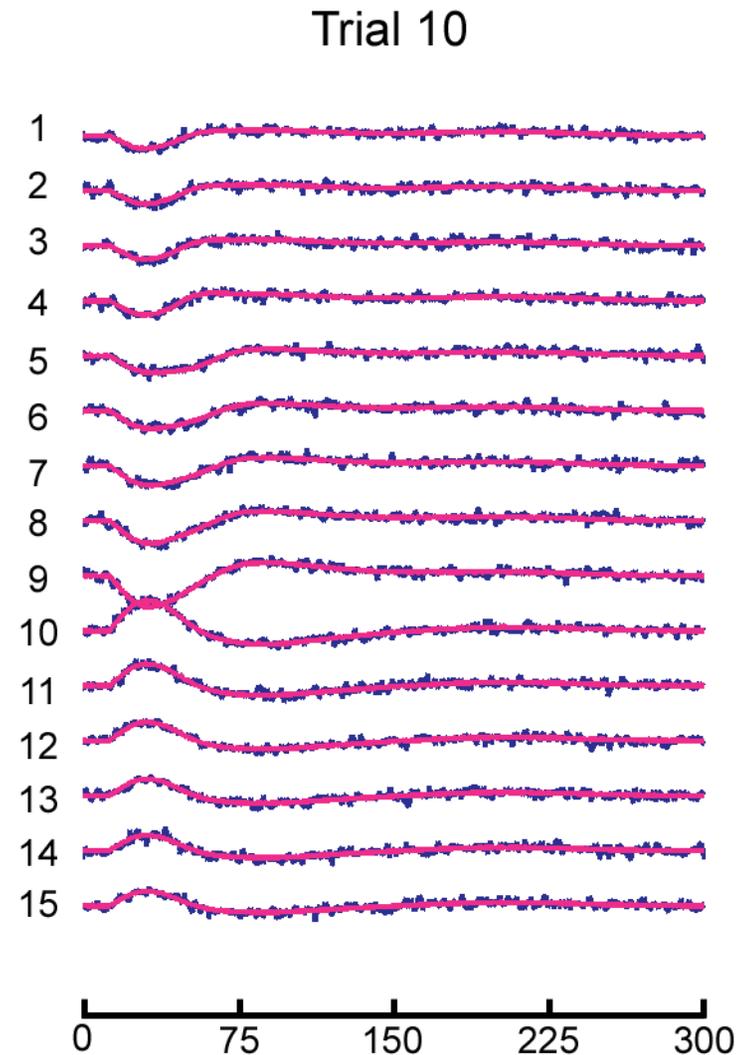
In this example the SNR is low.

All other simulations had higher SNR.

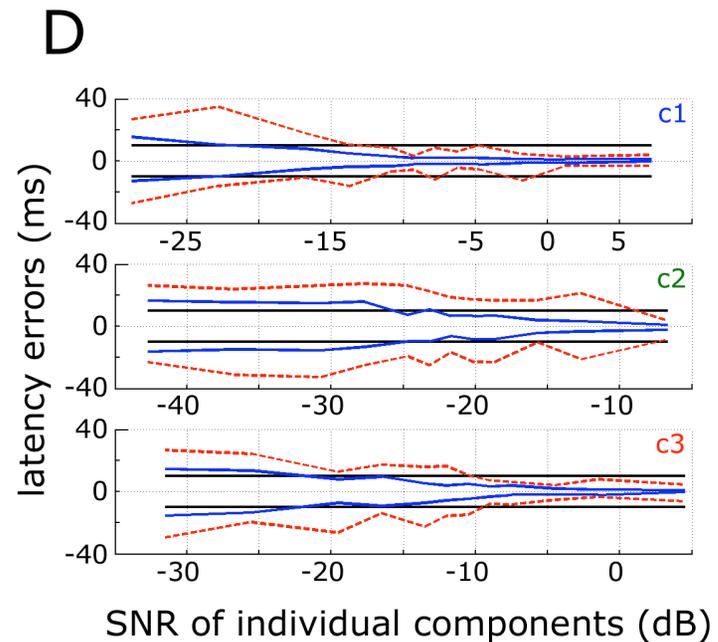
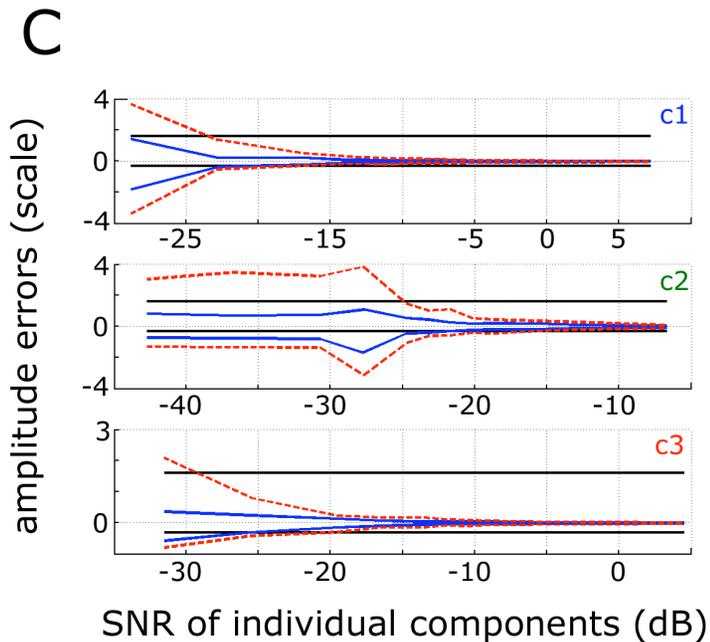
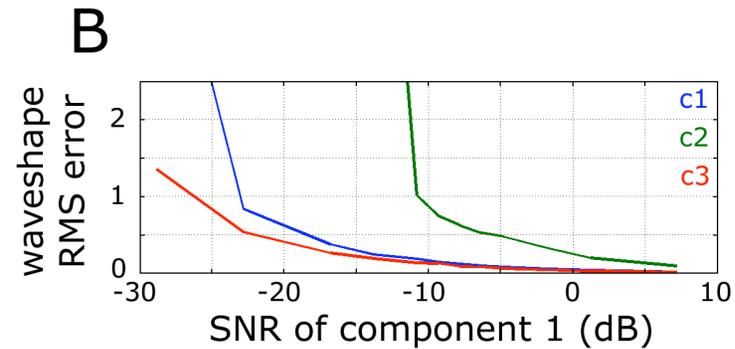
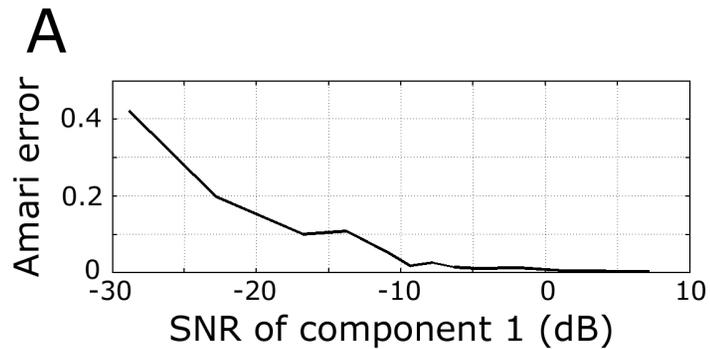


# Single-Trial Reconstructions

These are the synthetic trials with their reconstructions.

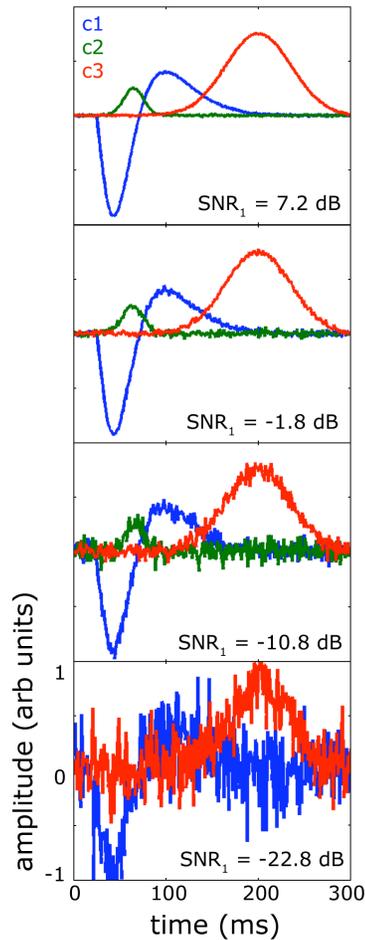


# Robustness to Noise

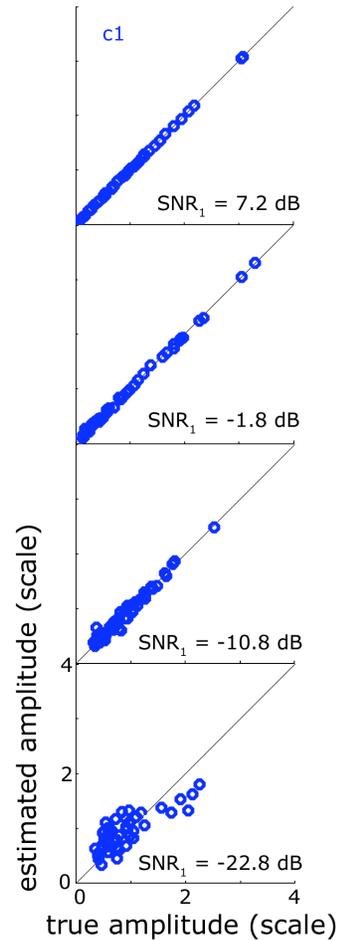


# Robustness to Noise

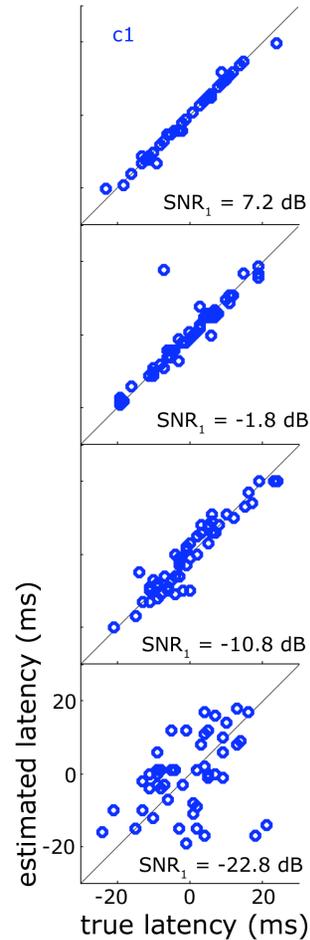
E



F



G



**Signal : Noise**

5 : 1 (5.28)

2 : 3 (0.67)

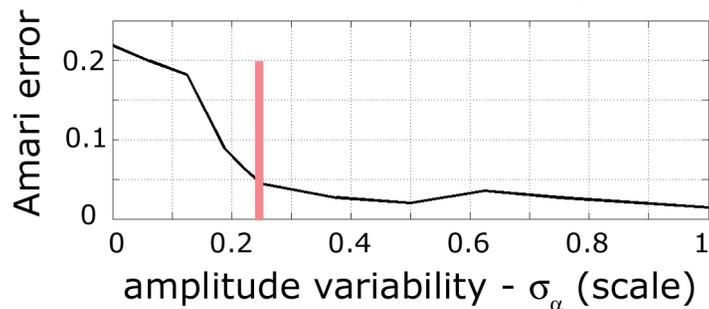
1 : 12 (0.084)

1 : 189 (0.0053)

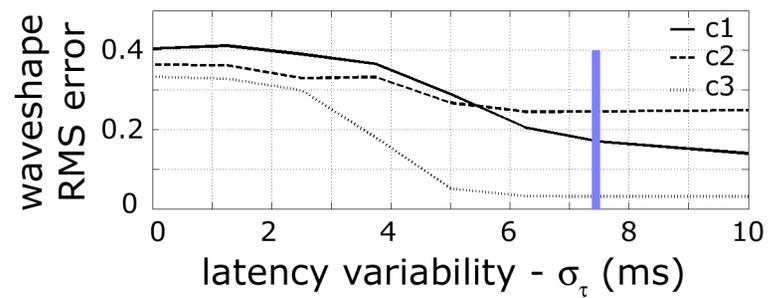
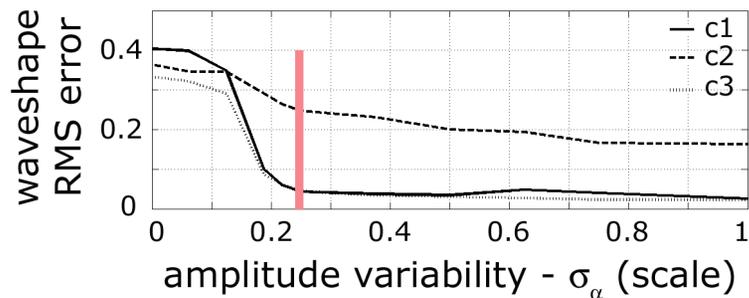
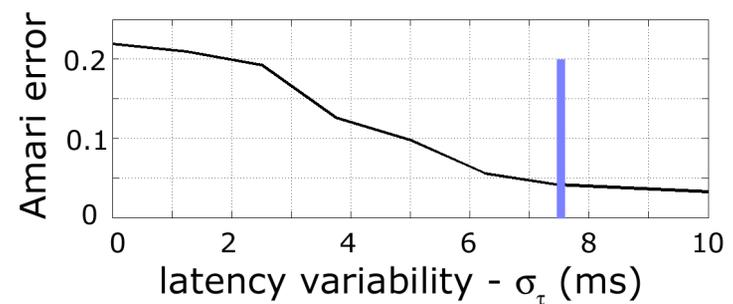
# Dependence on Variability

Estimation error decreases with increasing variability

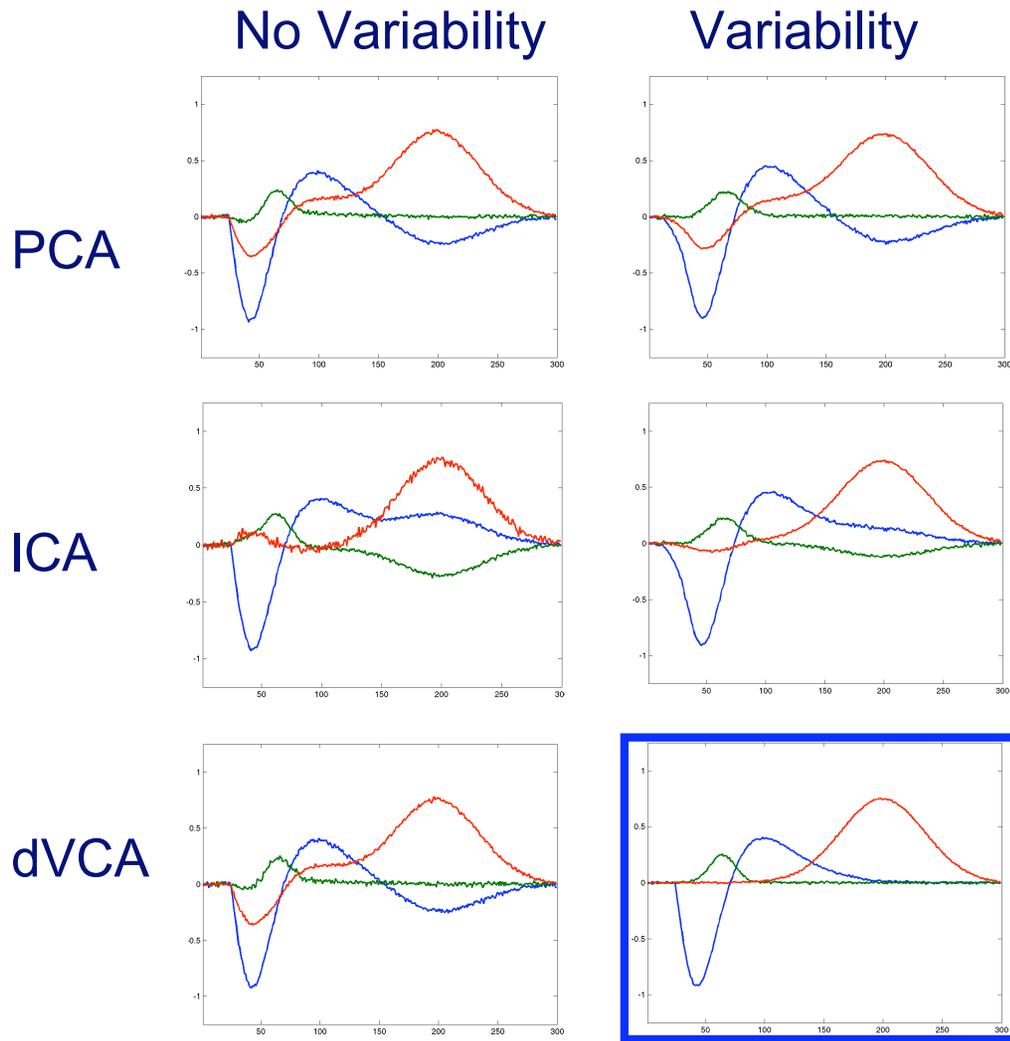
Amplitude Variability



Latency Variability

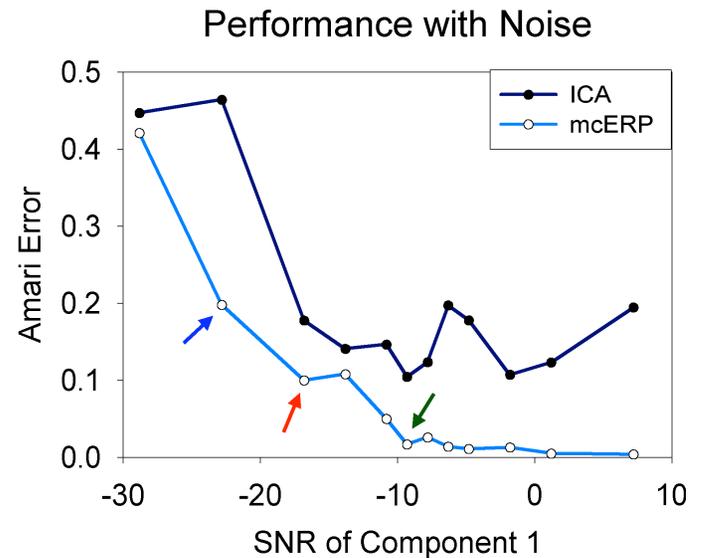
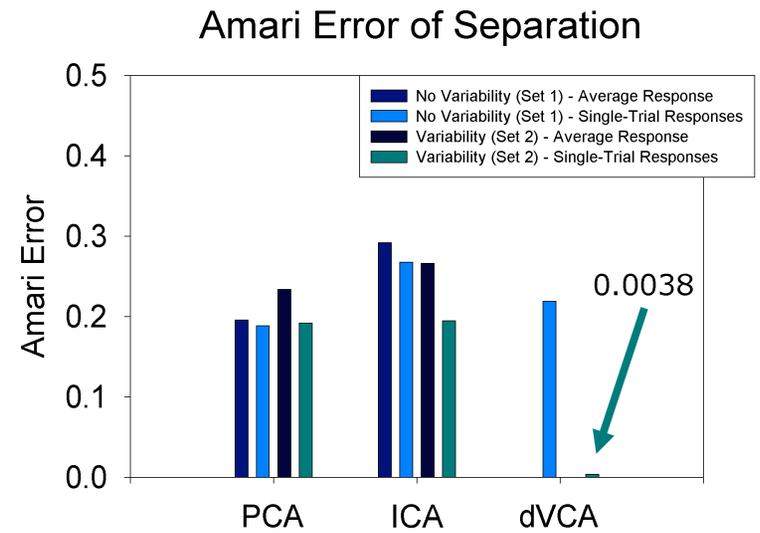


# Relation to PCA and ICA



28 March 2004

EPIC XIV



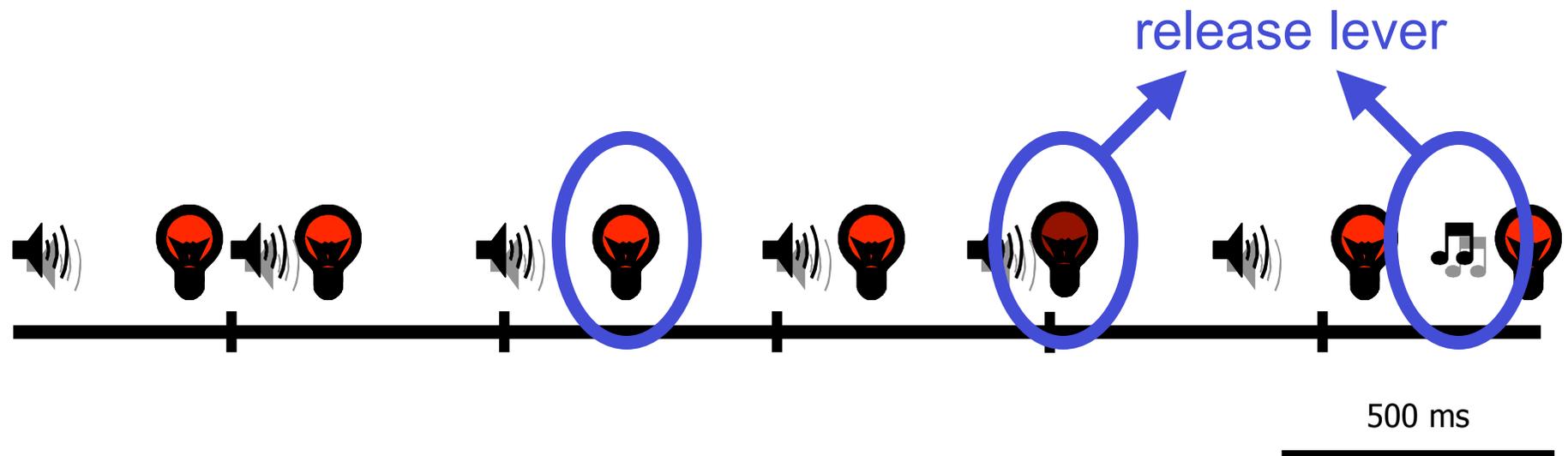
# Experimental Data

# Application to Real Data

Subject: Male macaca fascicularis

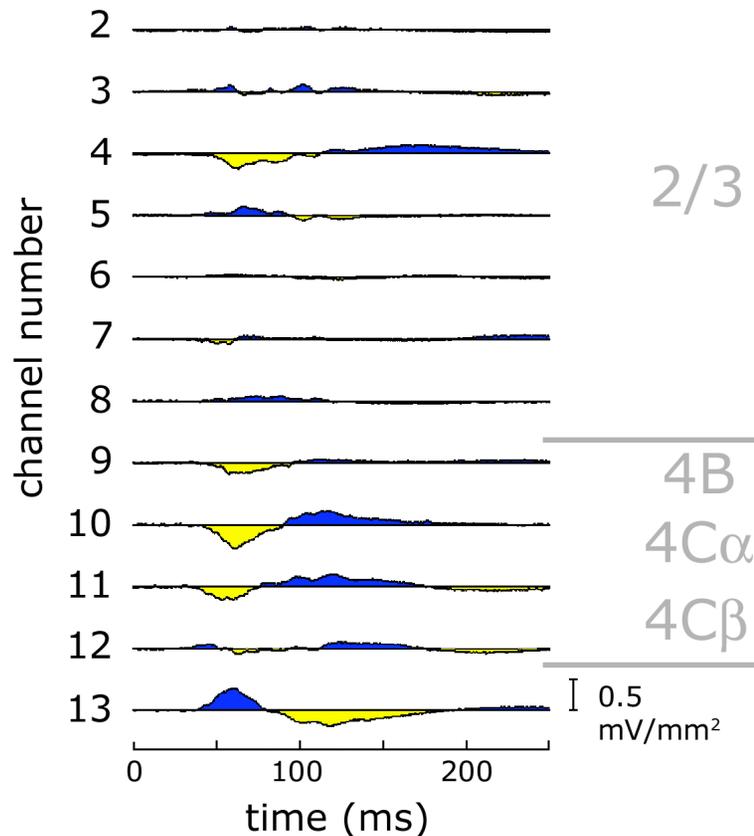
Task: Intermodal selective attention task

Mehta AD, et al. 2000 Cereb Cortex 10(4)

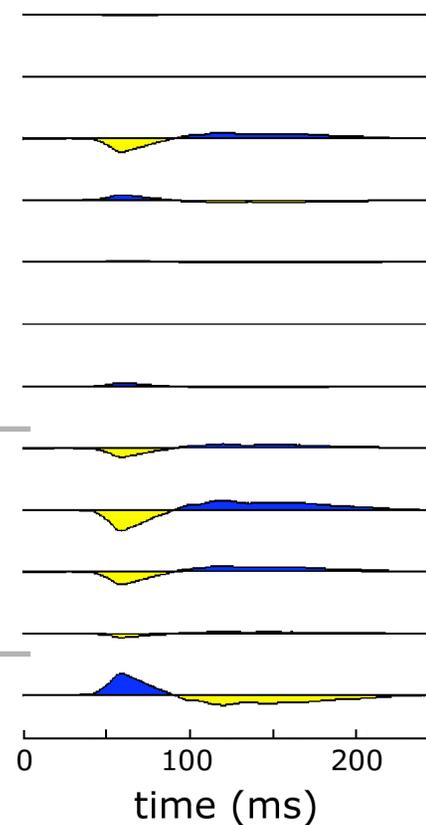


# Estimating a Single Component

CSD of the Average ERP



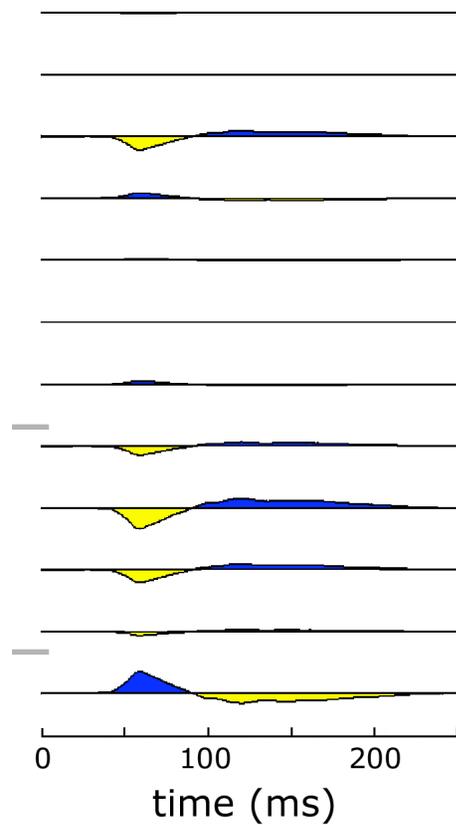
CSD of Component 1



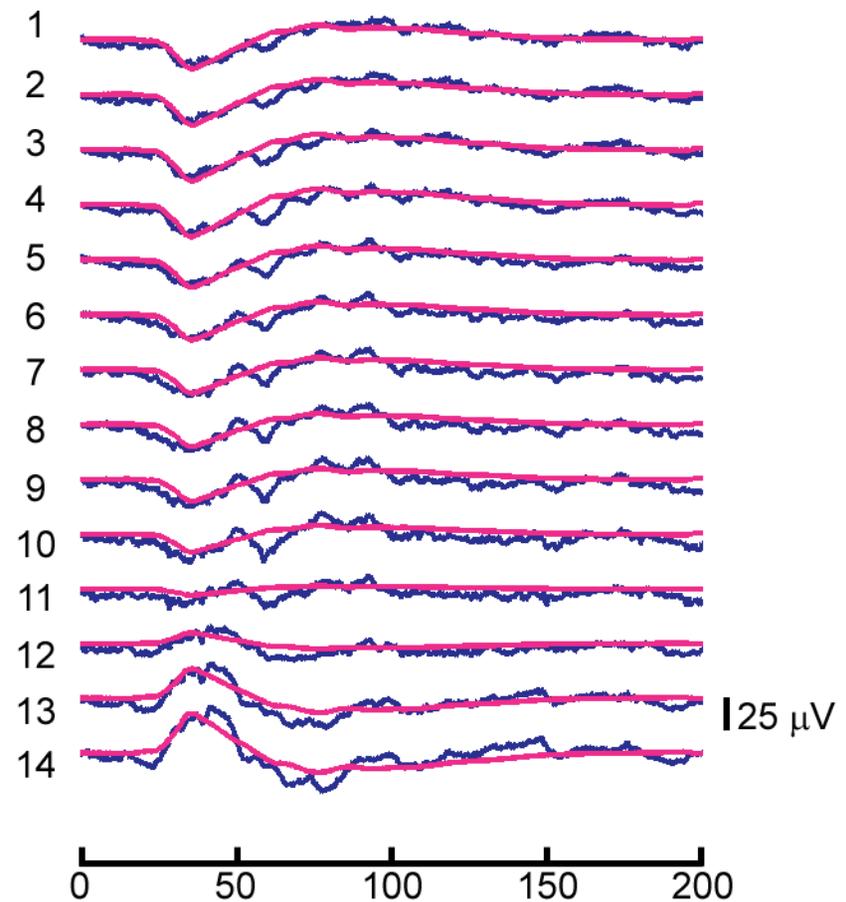
Not surprisingly, the single component model captures much of the character of the average.

# How Reasonable is the Model?

CSD of Component 1

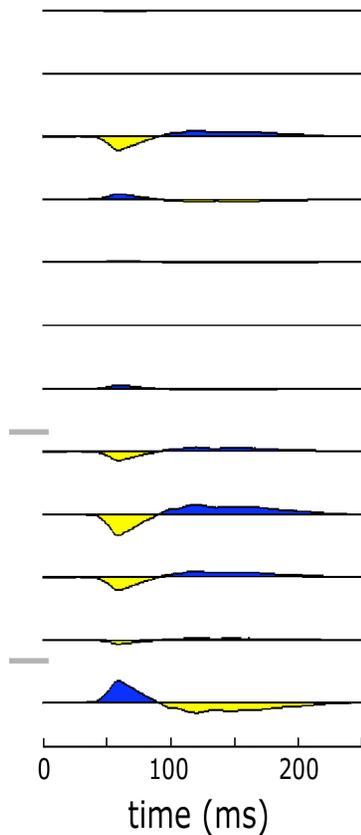


Trial 10

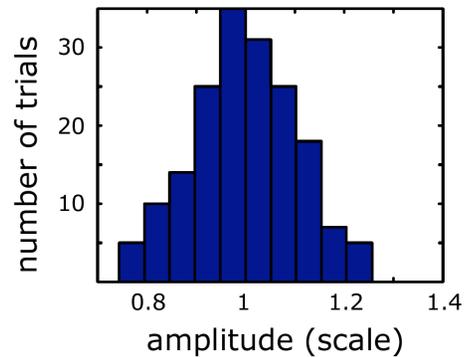


# Single-Trial Characteristics

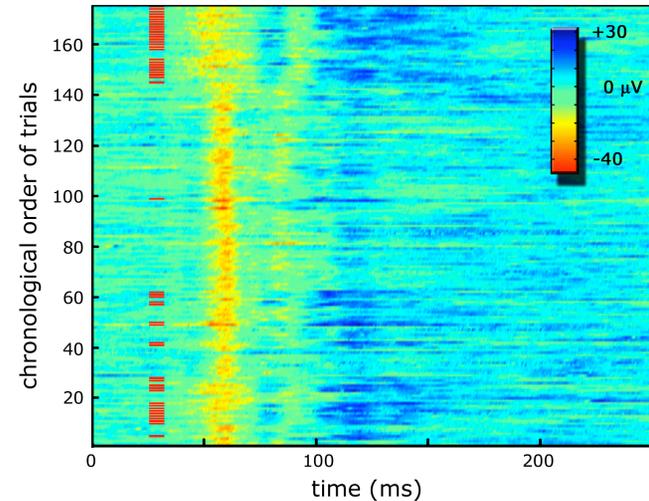
Component 1  
CSD



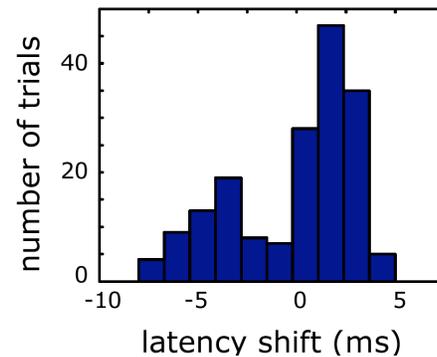
Single-Trial  
Amplitudes



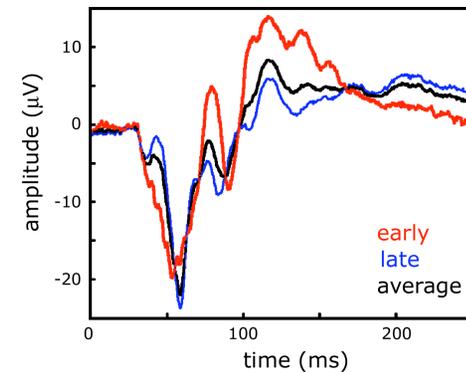
Individual Trials



Single-Trial  
Latencies

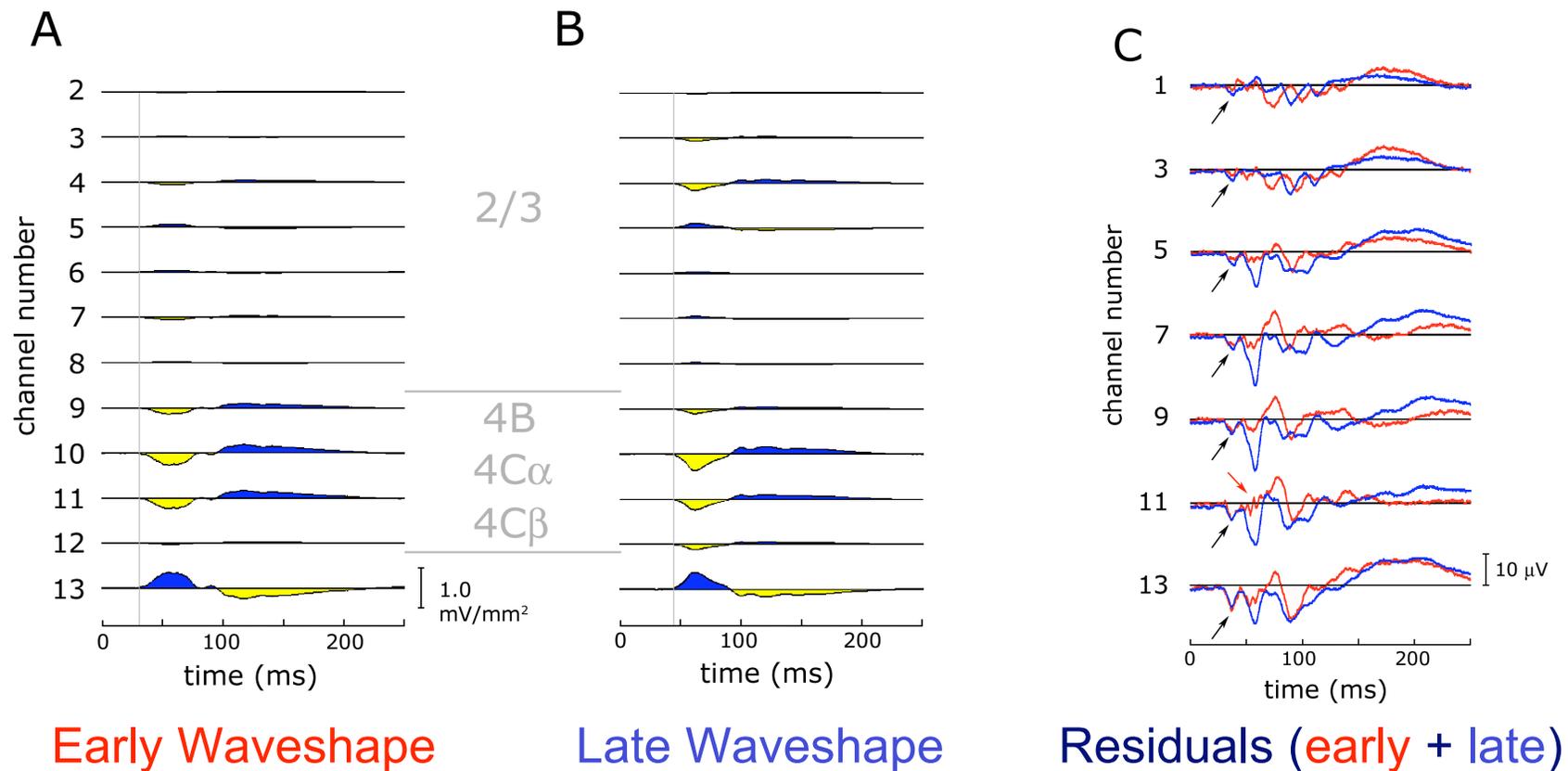


Early and Late  
Subaverages



# Splitting the Dataset

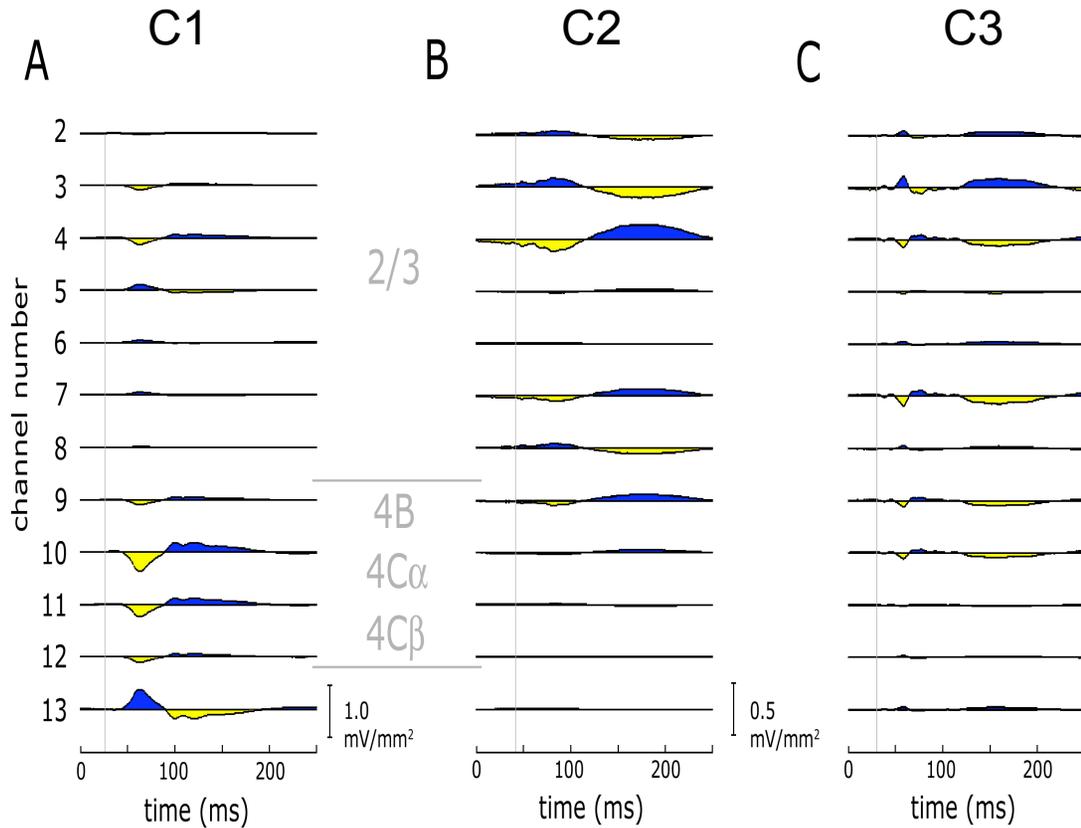
Splitting the dataset reveals the way in which the two response states differ



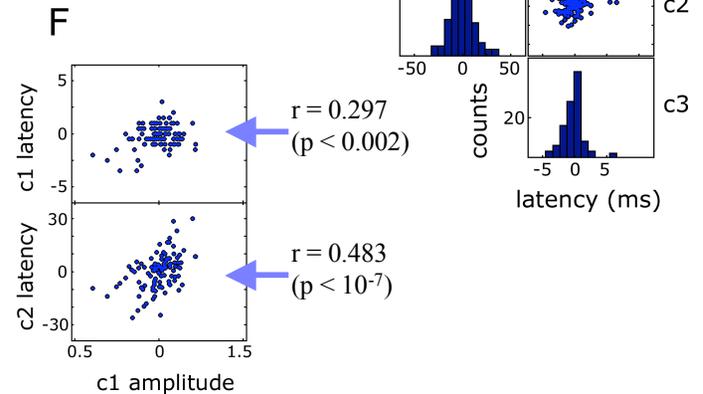
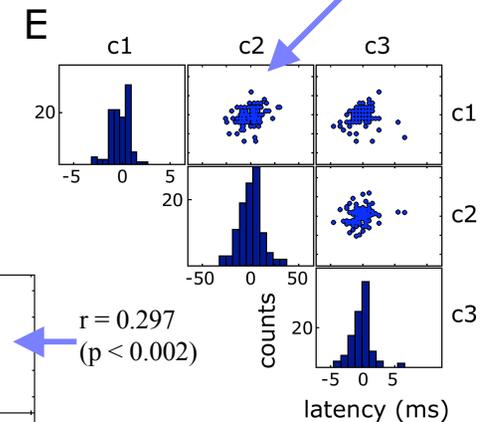
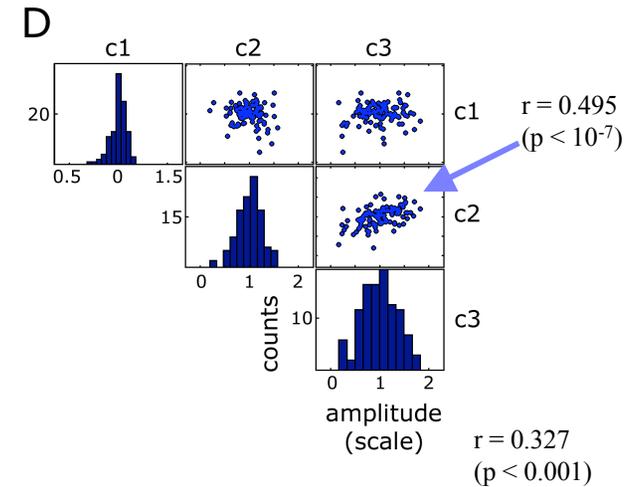
28 March 2004

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# Three Components from the Late Subset



C1 and C2 are coupled in latency  
 C2 and C3 are coupled in amplitude  
 C1 amplitude and C1 latency are coupled

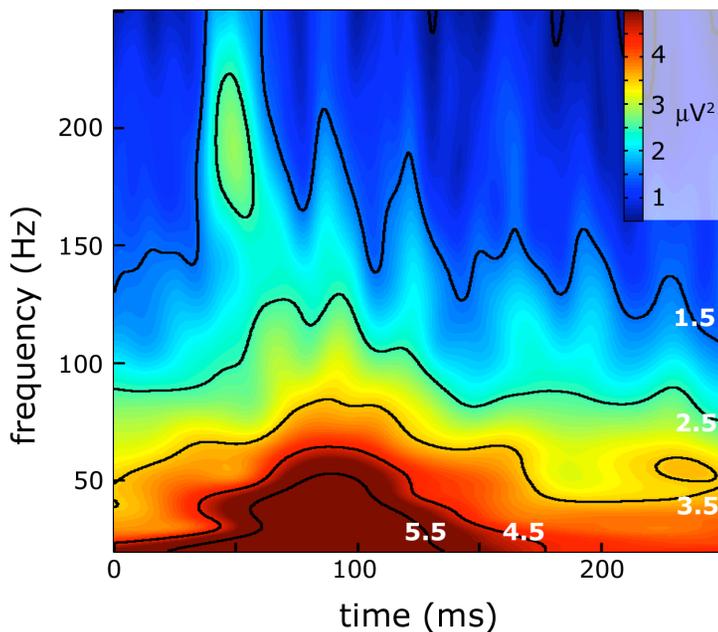


# Studying Single-Trial Oscillations

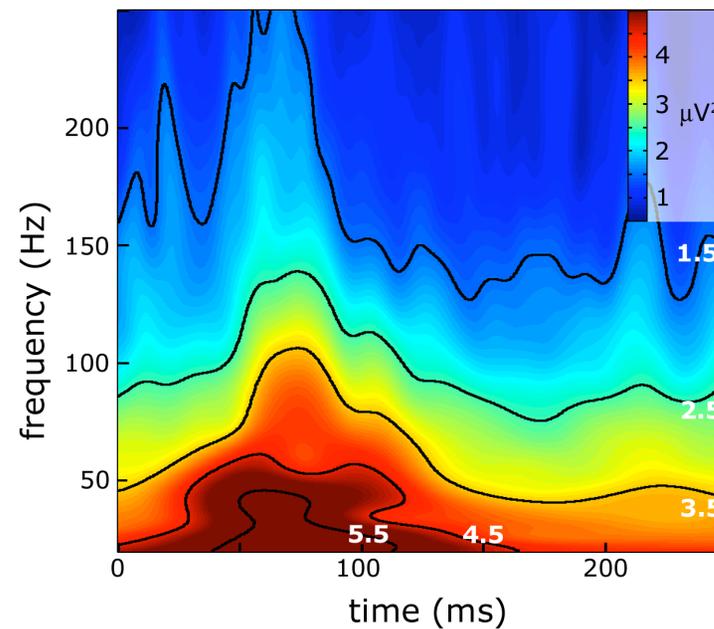
We have developed some new techniques to study oscillatory bursts, which we have found are present over a wide range of frequencies.

For example, the early and late subsets show a difference in the presence of 160-220 Hz bursts between 42-57 ms.

Early Subset



Late Subset

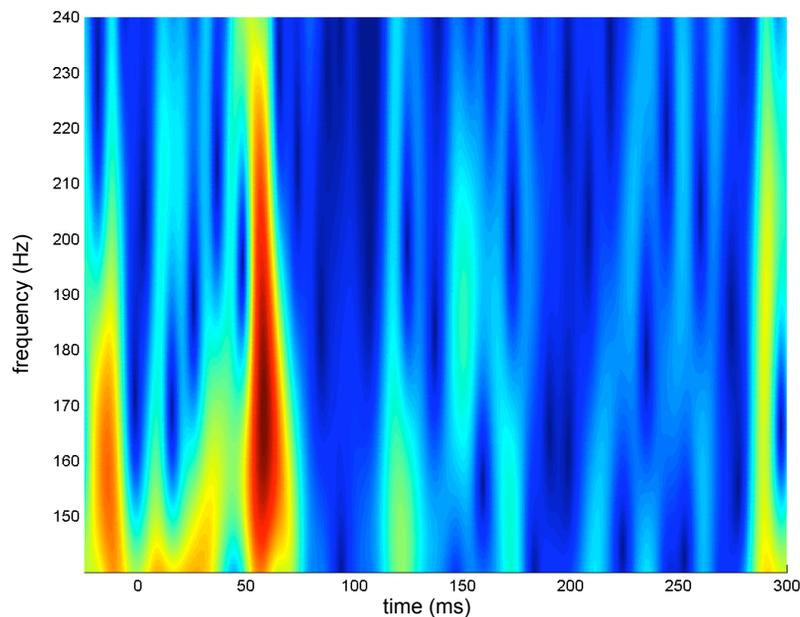


# Characterizing Oscillatory Bursts

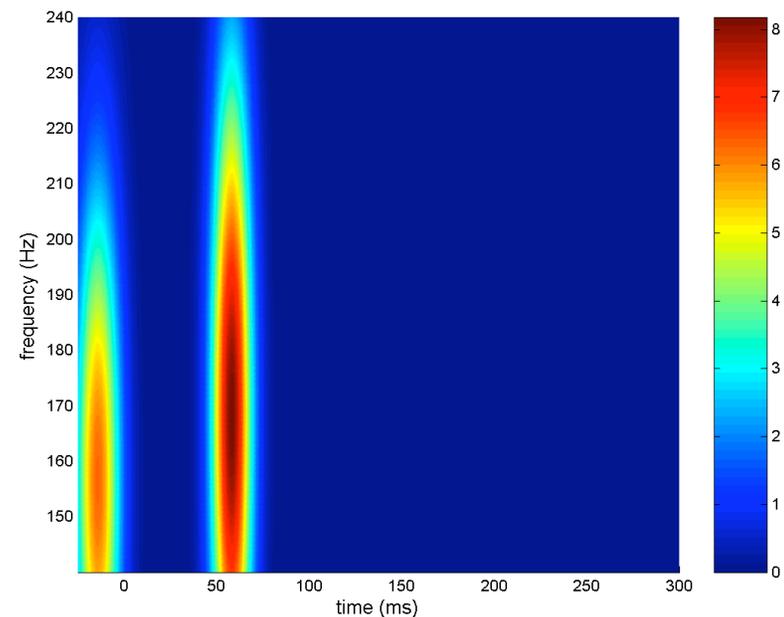
Using dVCA to identify components in the single trial allows us to remove them from the data, leaving unmodeled ongoing activity.

We can then identify **oscillatory bursts** and characterize them.

Time-Frequency plot of Early Subset



Modeled Bursts in the same trial



# Ongoing Challenges

**dVCA is not fast.** There is a lot of computation that goes on. The more data and channels, the slower the analysis gets. High dimensional spaces are notoriously difficult to search. Advances in optimization are extremely relevant to this work.

## **Difficulties with large numbers of components.**

The danger of getting trapped in local solutions of the model space becomes greater with increasing numbers of components. We have found that it is best to use dVCA little by little to learn about the signals.

## **Waveshape changes.**

We do not accommodate waveshape changes. Although we have some ideas that we are currently working on to solve this.



## Collaborators

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NASA Aerospace Technology Enterprise (KHK)

NARSAD Young Investigator Award (KHK)

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## Thanks

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Extras

## Obtaining the Marginal Posterior

Marginalizing over the variance we get the marginal posterior

$$p(\mathbf{C}, \mathbf{s}(t), \hat{\mathbf{a}}, \hat{\mathbf{o}} | \mathbf{x}(t), I) \propto Q^{\frac{-MRT}{2}} p(\mathbf{C} | I) p(\mathbf{s} | I)$$

Like before we look at the Log probability

$$\log P = -\frac{MRT}{2} \log Q + \text{const}$$

From here we look at the derivative of the Log probability wrt each of our model parameters. Setting the derivative equal to zero gives us the most probable solution.

## Estimating the Waveshapes

For the  $j^{\text{th}}$  component at time  $q$ , we have

$$\frac{\partial \log P}{\partial s_j(q)} = -\frac{MRT}{2} Q^{-1} \frac{\partial Q}{\partial s_j(q)} = 0$$

with

$$\frac{\partial Q}{\partial s_j(q)} = -2 \sum_{m=1}^M \sum_{r=1}^R \left[ W C_{mj} \alpha_{jr} - (C_{mj} \alpha_{jr}) \right] s_j(q)$$

where

$$W = x_{mr}(q + \tau_{jr}) - \sum_{\substack{n=1 \\ n \neq j}}^N C_{mn} \alpha_{nr} s_n(q - \tau_{nr} + \tau_{jr})$$

$$\hat{s}_j(q) = \frac{\sum_{m=1}^M \sum_{r=1}^R W C_{mj} \alpha_{jr}}{\sum_{m=1}^M \sum_{r=1}^R (C_{mj} \alpha_{jr})}$$

## Estimating the Amplitudes

For the  $j^{\text{th}}$  component during the  $p^{\text{th}}$  trial

$$\hat{\alpha}_{jp} = \frac{\sum_{m=1}^M \sum_{t=1}^T [U V]}{\sum_{m=1}^M \sum_{t=1}^T V^2}$$

where

$$U = \left( x_{mp}(t) - \sum_{\substack{n=1 \\ n \neq j}}^N C_{mn} \alpha_{np} s_n(t - \tau_{np}) \right)$$

$$V = C_{mj} s_j(t - \tau_{jp})$$

## Estimating the Source-Channel Couplings

For the coupling between the  $j^{\text{th}}$  component and the  $i^{\text{th}}$  channel

$$\hat{C}_{ij} = \frac{\sum_{r=1}^S \sum_{t=1}^T [X Y]}{\sum_{r=1}^S \sum_{t=1}^T Y^2}$$

where

$$X = \left( x_{ir}(t) - \sum_{\substack{n=1 \\ n \neq j}}^N C_{in} \alpha_{nr} s_n(t - \tau_{nr}) \right)$$

$$Y = \alpha_{jr} s_j(t - \tau_{jr})$$

## Estimating the Latency

For the latency of the  $j^{\text{th}}$  component during the  $p^{\text{th}}$  trial we **maximize**

$$Z = \sum_{m=1}^M \sum_{t=1}^T \left[ C_{mj} \alpha_{jp} s_j(t - \tau_{jp}) \left( x_{mp}(t) - \sum_{\substack{n=1 \\ n \neq j}}^N C_{mn} \alpha_{np} s_n(t - \tau_{np}) \right) \right]$$

$$\hat{\tau}_{jp} = \arg \max Z$$

## The Iterative Algorithm

$$\hat{C}_{ij} = \frac{\sum_{r=1}^S \sum_{t=1}^T [X Y]}{\sum_{r=1}^S \sum_{t=1}^T Y^2}$$

$$\hat{\tau}_{jp} = \arg \max Z$$

$$\hat{\alpha}_{jp} = \frac{\sum_{m=1}^M \sum_{t=1}^T [U V]}{\sum_{m=1}^M \sum_{t=1}^T V^2}$$

$$\hat{s}_j(q) = \frac{\sum_{m=1}^M \sum_{r=1}^R W C_{mj} \alpha_{jr}}{\sum_{m=1}^M \sum_{r=1}^R (C_{mj} \alpha_{jr})}$$

0. Begin with initial guesses for the waveshapes  $\hat{s}(q)$

$$\alpha_r = 1$$

$$\tau_r = 0$$

1. Estimate the source-detector couplings  $\hat{C}_{ij}$

2. Estimate the single trial latencies  $\tau_p$

3. Estimate the single trial amplitudes  $\alpha_p$

4. Estimate the waveshapes  $\hat{s}(q)$

5. Go to step 1.